

New look at the $[70, 1^-]$ nonstrange and strange baryons in the $1/N_c$ expansion

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Abstract. The masses of excited nonstrange and strange baryons belonging to the multiplet $[70, 1^-]$ are calculated in the $1/N_c$ expansion to order $1/N_c$ with a new method which allows to considerably reduce the number of linearly independent operators entering the mass formula. This study represents an extension to SU(6) of our work on nonstrange baryons, the framework of which was SU(4).

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In the $1/N_c$ expansion method [1, 2], when SU(3) is broken, the mass operator takes the following general form, as first proposed in Ref. [3] for the symmetric baryon multiplet $[N_c]$

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \quad (1)$$

where O_i are formed of $SU(N_f)$ invariant operators combined with SO(3) and SU(2)-spin generators and B_i break SU(3) explicitly. Presently we are studying its applicability to orbitally excited states of symmetry $[N_c - 1, 1]$.

The standard approach to applying the $1/N_c$ expansion method to baryons described by mixed symmetric states $[N_c - 1, 1]$, both in the orbital and flavor-spin degrees of freedom, is to decouple the system of N_c quarks into a ground state core of $N_c - 1$ quarks and an excited quark [4]. This implies that each generator of $SU(2N_f)$ and SO(3) has to be written as a sum of two terms, one acting on the excited quark and the other on the core. As a consequence, the number of linearly independent operators O_i in the mass formula increases tremendously and the number of the coefficients c_i and d_i encoding the quark dynamics and the flavor symmetry breaking, to be determined in a numerical fit, becomes much larger than the experimental data available, as for example for the lowest negative parity nonstrange baryons [4] where $d_i = 0$. Accordingly, the choice of the most dominant operators in the mass formula becomes out of control which implies that important physical effects can be missed.

In a previous work [5] we have proposed a new method where the core + quark separation is avoided. Then we deal with $SU(2N_f)$ generators acting on the whole system and the number of linearly independent operators turns out to be considerably smaller than the number of

data. All these operators can be included in the fit to clearly find out the most dominant ones up to order $1/N_c$. The knowledge of matrix elements of $SU(2N_f)$ generators between mixed symmetric states $[N_c - 1, 1]$ is necessary.

In this approach we have first analyzed the nonstrange $[70, 1^-]$ multiplet where the algebraic work was based on Ref. [6] which provided the matrix elements of SU(4) generators in terms of isoscalar factors of SU(4), initially derived in the context of nuclear physics but quite easily applicable to a system of N_c quarks. In this way we have shown that in the mass formula the flavor (in this case the isospin) term becomes as dominant in Δ resonances as the spin term in N resonances. This means that the corresponding coefficients c_i in (1) have comparable values and contribute dominantly to the flavor-spin breaking. Note that the flavor operator was neglected in Ref. [4].

Due to this interesting physical implication, presently we extend the method of Ref. [5] to incorporate the strange baryons. This means that we need the matrix elements of the SU(6) generators between mixed symmetric $[N_c - 1, 1]$ states. According to the generalized Wigner-Eckart theorem described in Ref. [6] this amounts at finding the corresponding isoscalar factors. The algebraic work has been performed in two steps. First we have obtained the isoscalar factors of all SU(6) generators for symmetric $[N_c]$ states [7] and next the isoscalar factors for mixed symmetric $[N_c - 1, 1]$ states [8]. The latter work has been completed in Ref. [9]. This report represents a summary of Ref. [9].

The coefficients c_i and d_i of the mass formula (1) have been obtained using the experimental masses of nonstrange and strange baryons from PDG [10]. In the numerical fit we considered the 17 resonances with a sta-

TABLE 1. Operators and their coefficients in the mass formula obtained from numerical fits. The values of c_i and d_i are indicated under the heading Fit n (n=1,2,3), in each case.

Operator	Fit 1 (MeV)	Fit 2 (MeV)
$O_1 = N_c \mathbb{I}$	489 ± 6	489 ± 6
$O_2 = \ell^i s^i$	5 ± 6	6 ± 6
$O_3 = \frac{1}{N_c} S^i S^i$	129 ± 19	129 ± 19
$O_4 = \frac{1}{N_c} (T^a T^a - \frac{1}{12} N_c (N_c + 6))$	167 ± 12	165 ± 11
$O_5 = \frac{3}{N_c} L^i T^a G^{ia}$	10 ± 8	5 ± 3
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	9 ± 1	9 ± 1
$O_7 = \frac{1}{N_c^2} L^i G^{ja} \{S^j, G^{ia}\}$	-24 ± 34	
$B_1 = -\mathcal{S}$	146 ± 10	146 ± 15
$B_2 = \frac{1}{N_c} S^i G^{i8} - \frac{1}{2\sqrt{3}} O_3$	78 ± 69	74 ± 69
χ_{dof}^2	1.42	1.33

tus of four and three stars and two mixing angles for the nonstrange resonances N'_J and N_J ($J = 1/2, 3/2$) found in Ref. [11]. These states are defined by the relations

$$\begin{aligned} |N'_J\rangle &= \cos \theta_J |^4 N_J\rangle + \sin \theta_J |^2 N_J\rangle \\ |N_J\rangle &= \cos \theta_J |^2 N_J\rangle - \sin \theta_J |^4 N_J\rangle \end{aligned} \quad (2)$$

Experimentally one finds $\theta_{1/2}^{\text{exp}} \approx -0.56$ rad and $\theta_{3/2}^{\text{exp}} \approx 0.10$ rad [11]. The same reference gives the mixing matrix of the $\Lambda(S01)$ resonances in terms of the flavor singlet $^2 1$ and $^2 8$ and $^4 8$ octet components. The transformation is

$$\begin{pmatrix} \Lambda(1800) \\ \Lambda(1670) \\ \Lambda(1405) \end{pmatrix} = \begin{pmatrix} -0.17 & 0.89 & -0.43 \\ -0.95 & -0.04 & 0.30 \\ 0.25 & 0.46 & 0.85 \end{pmatrix} \begin{pmatrix} ^4 8 \\ ^2 8 \\ ^2 1 \end{pmatrix} \quad (3)$$

The output of the fit is shown in Table 1. Like for SU(4) we found that the spin and flavor operators O_3 and O_4 contribute nearly equally to the mass formula. For example in the Fit 1 they are $c_3 = 129 \pm 19$ MeV and $c_4 = 167 \pm 12$ MeV respectively and they contribute dominantly to the flavor-spin breaking. The matrix elements of O_3 and O_4 are of order $1/N_c$ except for O_4 in flavor singlets when they become of order $\mathcal{O}(N_c^0)$ [9]. The expression of O_4 was introduced in Ref. [12] where it was shown that it recovers the expectation value of the isospin operators when $N_f = 2$. The operators containing the angular momentum components have small coefficients indicating only a small SO(3) breaking. In the Fit 2 the operator O_7 , which has a rather complex form, is removed and the χ_{dof}^2 slightly improves. The SU(3) breaking is dominated by B_1 where \mathcal{S} is the strangeness.

The total mass and the partial contributions of the operators considered in the Fit 2 are indicated in Table 2. The total masses have been obtained by including the mixing of N_J and N'_J ($J = 1/2, 3/2$) from Eq. (2) and the transformation matrix of $\Lambda(S01)$ resonances defined by Eq. (3). It turns out that O_3 is dominant in the $^4 8$ octets and O_4 in the decuplets and the flavor singlets. In the latter case the quantity $c_4 O_4$ is large and negative giving to $\Lambda''_{3/2}$ a mass practically identical to the experimental value of $\Lambda(1520)$. However this contribution is not enough for $\Lambda''_{1/2}$ to be identified with $\Lambda(1405)$. The situation is similar to all constituent quark models where $\Lambda(1405)$ still raises serious problems having a mass at least 150 MeV above the experimental value [13].

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TABLE 2. The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion obtained from the Fit 2. The last two columns give the empirically known masses [10] and the resonance name and status .

	Part. contrib. (MeV)								Total (MeV)	Exp. (MeV)	Name, status
	$c_1 O_1$	$c_2 O_2$	$c_3 O_3$	$c_4 O_4$	$c_5 O_5$	$c_6 O_6$	$d_1 B_1$	$d_2 B_2$			
$N_{\frac{1}{2}}$	1467	-2	32	41	-5	0	0	0	1513 ± 22	1538 ± 18	$S_{11}(1535)^{****}$
$\Lambda_{\frac{1}{2}}$							146	-5	1648 ± 36	1670 ± 10	$S_{01}(1670)^{****}$
$\Sigma_{\frac{1}{2}}$							146	-5	1674 ± 14		
$\Xi_{\frac{1}{2}}$							292	-11	1815 ± 27		
$N_{\frac{3}{2}}$	1467	1	32	41	2	0	0	0	1542 ± 20	1523 ± 8	$D_{13}(1520)^{****}$
$\Lambda_{\frac{3}{2}}$							146	-5	1685 ± 12	1690 ± 5	$D_{03}(1690)^{****}$
$\Sigma_{\frac{3}{2}}$							146	-5	1685 ± 12	1675 ± 10	$D_{13}(1670)^{****}$
$\Xi_{\frac{3}{2}}$							292	-11	1825 ± 25	1823 ± 5	$D_{13}(1820)^{***}$
$N'_{\frac{1}{2}}$	1467	-5	162	41	-12	-18	0	0	1656 ± 22	1660 ± 20	$S_{11}(1650)^{****}$
$\Lambda'_{\frac{1}{2}}$							146	-27	1721 ± 36	1785 ± 65	$S_{01}(1800)^{***}$
$\Sigma'_{\frac{1}{2}}$							146	-27	1754 ± 34	1765 ± 35	$S_{11}(1750)^{***}$
$\Xi'_{\frac{1}{2}}$							292	-53	1873 ± 60		
$N'_{\frac{3}{2}}$	1467	-2	162	41	-5	15	0	0	1681 ± 20	1700 ± 50	$D_{13}(1700)^{***}$
$\Lambda'_{\frac{3}{2}}$							146	-27	1797 ± 29		
$\Sigma'_{\frac{3}{2}}$							146	-27	1797 ± 29		
$\Xi'_{\frac{3}{2}}$							292	-53	1916 ± 56		
$N_{\frac{5}{2}}$	1467	3	162	41	7	-4	0	0	1677 ± 14	1678 ± 8	$D_{15}(1675)^{****}$
$\Lambda_{\frac{5}{2}}$							146	-27	1796 ± 24	1820 ± 10	$D_{05}(1830)^{***}$
$\Sigma_{\frac{5}{2}}$							146	-27	1796 ± 24	1775 ± 5	$D_{15}(1775)^{****}$
$\Xi_{\frac{5}{2}}$							292	-54	1915 ± 52		
$\Delta_{\frac{1}{2}}$	1467	2	32	206	-10	0	0	0	1697 ± 18	1645 ± 30	$S_{31}(1620)^{****}$
$\Sigma'_{\frac{1}{2}}$							146	-5	1838 ± 20		
$\Xi'_{\frac{1}{2}}$							292	-11	1978 ± 31		
$\Omega_{\frac{1}{2}}$							437	-16	2119 ± 45		
$\Delta_{\frac{3}{2}}$	1467	-1	32	206	-10	0	0	0	1709 ± 19	1720 ± 50	$D_{33}(1700)^{****}$
$\Sigma'_{\frac{3}{2}}$							146	-5	1850 ± 18		
$\Xi'_{\frac{3}{2}}$							292	-11	1990 ± 29		
$\Omega_{\frac{3}{2}}$							437	-16	2131 ± 43		
$\Lambda''_{\frac{1}{2}}$	1467	-6	32	-124	0	0	146	-5	1547 ± 41	1407 ± 4	$S_{01}(1405)^{****}$
$\Lambda''_{\frac{3}{2}}$	1467	3	32	-124	0	0	146	-5	1519 ± 15	1520 ± 1	$D_{03}(1520)^{****}$
$N_{1/2} - N'_{1/2}$	0	-2	0	0	-55	-2	0	0	-55		
$N_{3/2} - N'_{3/2}$	0	-3	0	0	18	4	0	0	18		